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Teacher Reflection on a Teaching-Learning Situation Regarding the Concept of Relation in Secondary Education

Reflexión docente de una situación de enseñanza y aprendizaje sobre el concepto de relación en educación secundaria

Reflexão docente de uma situação de ensino e aprendizagem sobre o conceito de relação no ensino médio

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Resumen

Este estudio se enmarca en las investigaciones en didáctica de la matemática sobre la formación de docentes. Particularmente, se enfoca el conocimiento de esta población tomando como referente los planteamientos teóricos del modelo denominado conocimiento especializado del profesor de matemática (MTSK) y de la reflexión docente como competencia profesional. Corresponde a una investigación cualitativa descriptiva, basada en un estudio instrumental de casos, cuyo propósito es describir y analizar las manifestaciones de docentes de matemática de educación secundaria al observar una situación de enseñanza-aprendizaje que muestra a un profesor cuando enseña el concepto de relación. Para esto, se utilizaron las narrativas escritas por siete docentes de matemática de educación secundaria en servicio, las cuales señalan aspectos identificados como relevantes de la situación observada. El análisis de la información se realizó mediante la definición de categorías, subcategorías y unidades de análisis basadas en los componentes del modelo MTSK. Los resultados muestran que las personas docentes participantes acentúan, en las manifestaciones registradas en las narrativas, atributos asociados principalmente al conocimiento didáctico del contenido matemático. La reflexión de este grupo de docentes se caracteriza desde tres aspectos específicos del modelo MTSK: la enseñanza, el aprendizaje y el conocimiento de los temas. Se interpreta que la reflexión se ha visto influenciada por la experiencia profesional de las personas docentes participantes en el estudio.

Palabras clave: Conocimiento especializado del profesor de matemáticas; reflexión docente; relación; narrativas; educación secundaria.

Abstract

This study is part of the research in *didactics of mathematics* on teacher training. In particular, the study focuses on the knowledge of mathematics teachers, taking as a reference the theoretical approaches of the Mathematics Teacher's Specialized Knowledge (MTSK) model and teacher reflection as a professional

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competence. The study corresponds to a descriptive-qualitative research, based on an instrumental study of cases, whose purpose is to describe and analyze the manifestations of mathematics teachers of Secondary Education when they observe a teaching-learning situation that shows a teacher of mathematics teaching the concept of relation. For this, narratives written by seven Secondary Education mathematics teachers in service were used, which indicate some aspects identified by them as relevant from the observed situation. The analysis of the gathered information was carried out by defining categories, subcategories, and analysis units based on the components of the MTSK model. The results indicate that participating teachers accentuate, in their manifestations, attributes mainly associated with didactic knowledge of mathematical content. The reflection of this group of teachers is characterized by three specific aspects of the MTSK model: teaching, learning, and knowledge of the topics. It is interpreted that teacher reflection has been influenced by the professional experience of the teachers participating in the study.

Keywords: Mathematics teachers' specialized knowledge; Noticing; Relation; Narratives; Secondary Education.

Resumo

Este estudo se enquadra nas pesquisas em didática da matemática sobre a formação de docentes. Particularmente, está enfocado no conhecimento desta população tendo como referência as exposições teóricas do modelo denominado conhecimento especializado do professor de matemática (MTSK) e da reflexão docente como competência profissional. Corresponde a uma pesquisa qualitativa descritiva, baseada em um estudo instrumental de casos, cujo propósito é descrever e analisar as manifestações de docentes de matemática do ensino médio ao observar uma situação de ensino-aprendizagem que um professor mostra ao ensinar o conceito de relação. Para isso, foram utilizadas as narrativas escritas por sete docentes de matemática do ensino médio em serviço, que destacam aspectos identificados como relevantes da situação observada. A análise das informações foi realizada mediante a definição de categorias, subcategorias e unidades de análises baseadas nos componentes do modelo MTSK. Os resultados mostram que as pessoas docentes participantes destacam, nas manifestações registradas nas narrativas, atributos associados principalmente ao conhecimento didático do conteúdo matemático. A reflexão deste grupo de docentes se caracteriza a partir de três aspectos específicos do modelo MTSK: o ensino, a aprendizagem e o conhecimento dos temas. A interpretação é que a experiência profissional influenciou na reflexão das pessoas docentes participantes no estudo.

Palavras-chave: Conhecimento especializado do professor de matemáticas; reflexão docente; relação; narrativas; ensino médio.

Introduction

Recently, various investigations on the knowledge of the mathematics teacher have pointed out the importance of having a strong knowledge of the mathematical content, complemented with a particular knowledge for the teaching of that content.

Particularly, Climent, Romero-Cortés, Carrillo, Muñoz-Catalán y Contreras (2013) point out the existence of a plurality of perspectives and conceptualizations from which it is possible to approach the knowledge of the mathematics teacher; their studies present the model called *Mathematics Teacher's Specialized Knowledge* (hereinafter, MTSK). MTSK is considered as the "'ideal' specialized

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knowledge that a mathematics teacher requires for the management of the teaching process, with the objective, among others, that their students understand the why of things" (Galleguillos, Ribeiro y Montes, 2015, p. 2).

But, how can this knowledge be identified in mathematics teachers? Llinares (2016) emphasizes that the way the teacher knows and uses his/her knowledge affects the ways that he/she implements to demonstrate that mathematical knowledge to his/her students and the flexibility that he/she shows to answer their questions or queries. That is, the analysis of teacher behavior in practice can give samples or indicators of this specialized knowledge required for teaching mathematics. One way to analyze this behavior is through teacher reflection on educational practice.

According to Dolk (1997), the performance of the profession is benefited when reflection on various class experiences and the origin of the teacher's behavior in practice is promoted; this, based on the systematic structure that provides reflection from the cognitive, emotional, volitional, and behavioral dimensions, and when the stimulus to meta-reflection and reflective learning among peers are enhanced. This benefit can be attributed to the knowledge and performance of the person who reflects; that is, of teachers who reflect on their own practice or of those who reflect on the practice of other teachers. From this, we highlight the reflection that can be made of certain class experiences from the cognitive dimension (of the professional knowledge of the mathematics teacher), oriented to reflective analysis among peers.

In the particular case of this article, a class experience is considered to be a lesson in which a mathematics teacher presents a task—hereinafter, the observed teacher—to introduce the concept of mathematical relationship, to a group of students from Secondary Education. It is desired to emphasize the aspects that are relevant for a group of mathematics teachers who reflect on the performance of the observed teacher.

In this context, we ask ourselves: what aspects does a group of mathematics teachers point out when they analyze the recording of a mathematics lesson? What aspects does this group of teachers emphasize when they reflect on the performance of the observed teacher who teaches mathematics? Taking as a starting point the teaching experience of those who participate in the study, we are interested in recognizing those aspects related to mathematical content knowledge and the didactic knowledge of mathematical content that these teachers identify when looking at a teaching situation on the concept of relationship.

The purpose of this article is to show, based on an exploratory study, the mathematical and didactic aspects of teacher performance that a group of mathematics professors identifies when looking at a segment of a recording of a class, in which a teacher of this subject introducing the concept of relationship by solving an authentic task.

Theoretical Framework

The theoretical references of this research are based on the proposals on the knowledge model called *Mathematics Teacher's Specialized Knowledge* and *Teacher Reflection*. Both notions intertwine to offer a frame of reference for analyzing mathematics teaching and learning situations.

Mathematics Teacher's Specialized Knowledge

In the field of education, the knowledge of teachers is conceived as knowledge

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and experiences that they use with the purpose of producing learning and growth in students (Climent, 2002). In this sense, the way in which the teacher knows and uses the knowledge he possesses affects how he teaches the contents to the students, how he answers their questions and how it helps them to relate mathematical ideas to each other (Llinares, 2016).

Shulman (1986, 1987) highlights the need for teachers to have both knowledge of the content they teach, as well as knowledge that enables them to teach that content. The first provides teachers with tools to know precisely the definitions, it also allows them to justify propositions and learn about applications of the content in other disciplines; while the second allows them to select the most appropriate way to present the content to the students, while considering their abilities and limitations, as well as the diversity of their interests.

In this study, the knowledge of mathematics teachers is addressed from the MTSK model proposed by Carrillo, Contreras, Climent, Escudero-Ávila, Flores-Medrano y Montes (2014). MTSK is a theoretical proposal that models the core knowledge of the professional mathematics teachers and is also a methodological tool that allows the analysis of teaching practice through its different categories. This knowledge model maintains the separation proposed by Shulman (1986) into two knowledge domains: content knowledge and content didactic knowledge, particularized to mathematics. It stands out because he considers the specialization of teachers in each of the subdomains of knowledge, due to this, he conceives the specialized character as the intersection of specific mathematical and didactic knowledge for mathematics teachers. In this sense, specialized is understood as "any knowledge of a mathematical nature that the teacher may require in their professional work" (Escudero *et al*., 2015, p. 57). In addition, it is distinguished from other knowledge models of mathematics teachers because it proposes a system of categories and subcategories for each of these knowledge domains, which makes it useful as an analytical model to interpret and characterize the specialized knowledge of mathematics teachers. The MTSK also proposes the subdomain on beliefs and conceptions towards mathematics.

The characterization of the MTSK is detailed, focusing on the subdomains of mathematical content knowledge and pedagogical knowledge of mathematical content. Regarding the subdomain of beliefs and conceptions about mathematics, this has not been considered for the present investigation.

Mathematical Knowledge

Mathematical Knowledge (MK) is defined as "the teacher's knowledge of mathematics as a scientific discipline in a school context" (Escudero *et al*., 2015, p. 71). Different knowledge models converge in pointing out that there is a difference between the way of knowing the mathematical content by the teaching person and that of other users of the discipline, such as an engineer (Flores-Medrano, Escudero-Ávila, Aguilar, Carrillo, 2014). Its importance is due to the fact that teachers with a strong mathematical knowledge have greater tools to achieve that their students understand the content significantly and, in addition, they demonstrate a deep knowledge of mathematics when they offer different points of view on mathematical topics.

In MTSK, three subdomains are considered: *Knowledge of Topics* (KOT), *Knowledge of the Structure of Mathematics*

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(KSM), and *Knowledge of Practices in Mathematics* (KPM).

Regarding KOT, it is based on the premise that teachers must know the mathematical content that they teach their students. It considers what and how well the mathematics teacher knows the content to be taught and refers to mathematical content knowledge and its meanings in an informed manner. It is made up of the content that students will learn at a higher level of detail. Topics are understood as the contents coming from the blocks of knowledge traditionally considered in school mathematics. In the case of the Costa Rican curriculum: numbers, relationships and algebra, geometry, measurements, and statistics and probability.

Five aspects that characterize this knowledge are considered: phenomenology, properties and their foundations, representation registers, definitions, and procedures. Phenomenology is the knowledge of teachers about situations associated with the topic, as well as its uses and applications. Knowledge of properties and their foundations refers to knowledge about the mathematical argumentation of the attributes or qualities of a concept or procedure. Representation registers correspond to the knowledge about the different ways in which a concept can be made present, including the notation and the mathematical language linked to said representations. The knowledge of definitions refers to the set of properties that define a mathematical object, it also considers the alternative ways that teachers use to define it. Finally, procedural knowledge considers the knowledge of traditional and alternative algorithms, the necessary and sufficient conditions and the arguments of a mathematical nature that allow their use, as well as the peculiarities of the mathematical object that results when applying the procedure.

KSM is the knowledge of the relationships between different contents of the same level or of different educational levels. It exclusively emphasizes the connections between mathematical topics. Four categories of mathematical connections are proposed. Complexity connections correspond to knowledge about the ways in which the currently taught content is related to future content, while simplification connections refer to knowledge about the way the currently taught content is related to past content. Transversal content connections refer to a common quality possessed by two or more contents that relates them, among them, the modes of thought associated with said content. Finally, auxiliary connections are linked to knowledge about the use of concepts as tools for the development of others.

Regarding KPM, it is the knowledge about the ways of proceeding in mathematics, as well as the characteristics of mathematical work.

> It is about knowing how knowledge is explored and generated in mathematics, how relationships, correspondences and equivalences are established, how it is argued, reasoned, and generalized, what role the agreement has, and what characteristics are possessed by some of the elements with which mathematics is performed, such as a definition or a demonstration. (Flores-Medrano, Escudero-Ávila, Aguilar, Carrillo, 2014, p. 77-78)

This subdomain is made up of two categories: practices linked to mathematics in general and practices linked to a topic in mathematics. The first is a type of knowledge used to work in a generic way

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in mathematics, regardless of the concept addressed, while the second category is knowledge about ways of proceeding in a particular mathematical content.

Didactic Knowledge of Mathematical Content

Pedagogical Content Knowledge (PCK) is the knowledge of teachers about aspects related to mathematical content as an object of teaching and learning. It is characterized by being knowledge attributed to the teaching practice, which is why it is directly linked to the teaching staff. It conceives the mathematical content from three points of view: as a content to be taught, as a content to be learned and as a component of the learning standards that are intended to be achieved. These three emphases generate the three subdomains of this knowledge domain: *Knowledge of Mathematics Teaching* (KMT), *Knowledge of Features of Learning Mathematics* (KFLM), and *Knowledge of Mathematics Learning Standards* (KMLS). The aspects that characterize them are detailed below.

KMT refers to the knowledge about resources, materials, and strategies to present the content to the students, as well as the potential that these have in the teaching and learning processes. It also considers knowledge of tasks, conceived as learning opportunities. This subdomain is made up of three categories: (a) personal or institutionalized teaching theories, which corresponds to knowledge about specific teaching theories for the teaching of mathematics; also, it considers teaching strategies from personal experience of the teaching staff such as the use of analogies or metaphors, acquired from their professional experience; (b) material and virtual resources associated with the content to be taught, considers the knowledge of the benefits and difficulties associated with its use in the teaching of mathematical content, and (c) activities, tasks, examples and aids, which refers to knowledge on the potential that a task, example or activity may have, as well as when and what kind of help to offer the students during their learning.

Regarding KFLM, it corresponds to the knowledge about the learning characteristics of the mathematical content; it focuses on mathematical content as a learning object, so its interest is the learning characteristics caused by the interaction of the student with the mathematical content and not the characteristics of the student itself. It is comprised of four categories: (1) forms of learning, (2) strengths and difficulties associated with learning, (3) forms of student interaction with mathematical content, and (4) student conceptions of mathematics.

The forms of learning are related to the possible forms of apprehension linked to the nature of the mathematical content and consider the knowledge of personal or institutionalized theories about the cognitive development of the student, both for mathematics in general and for a particular content. The strengths and difficulties associated with learning refer to the knowledge about errors and difficulties attributed both to mathematics in general and to a particular content. The knowledge about the forms of interaction of the students with the mathematical content is associated with the knowledge about procedures and strategies, both typical and unusual, used by the students, as well as the language they use when manipulating the mathematical content. Finally, the students' conceptions of mathematics reveal their expectations and interests towards mathematics.

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Mathematics learning standards correspond to the knowledge about the conceptual level that the student is expected to learn at a given school moment. This subdomain considers three categories: (a) mathematical contents that are required to be taught, which is the knowledge about the mathematical contents that must be taught in a specific school grade; (b) expected level of conceptual and procedural development, which corresponds to knowledge about the degree of detail with which a content should be taught at a given school moment, and (c) sequencing of various topics, which accentuates knowledge about what the student must or can learn at a given school moment or one before or after this.

In summary, the MTSK model considers the specialization of knowledge of teachers as knowledge that only makes sense for them and that makes sense when they develop their teaching work.

Competence of teacher reflection

Teaching reflection is conceived as the professional competence that allows the identification of relevant mathematical aspects in a teaching and learning situation, interpreting them from a theoretical reference and making changes to improve them. In this sense, this teaching competence is linked to three skills: (1) identifying outstanding events that arise in the teaching and learning processes, (2) reasoning about interactions in the classroom based on the knowledge of the context in which teaching is done and (3) connecting specific classroom events with more general principles and ideas of teaching and learning processes (Sherin, Jacobs y Philipp, 2011).

In the literature, different terms can be found to refer to teacher reflection, among which we highlight "Noticing" (Mason, 2002) and "Professional noticing" (the term used by the cited authors is "Mirar profesionalmente") (Llinares, 2012, 2013, 2016).

This teaching competence allows mathematics teachers to analyze educational practices in a professional manner, distinguishing themselves from those who are not professionals in this area (Mason, 2002). The professional character is due to the use of knowledge to identify, interpret and offer options for improvement on outstanding aspects of a situation related to the teaching and learning of mathematics. In this sense, Llinares (2016) states:

> This way of posing the relation between the cognitive actions of identifying, registering, and interpreting particular aspects of mathematics teaching and learning situations makes more explicit the need to consider the role that mathematical knowledge and mathematics didactics possess as references for the observation and interpretation of the events identified and described. (p. 218)

Due to the above, the competence of teacher reflection is a demonstration of the use of mathematical content knowledge and of the didactic knowledge of mathematical content exhibited by the teacher (specialized knowledge) in teaching situations, emphasizing the link between professional knowledge and contexts of its use (Ball, Thames y Phelps, 2008; Hill, Ball y Schilling, 2008; Sherin, 2001; Sherin, Jacobs y Philipp, 2011).

Another aspect that characterizes the competence of teacher reflection is that its development does not take place spontaneously in teachers, but it is necessary to develop competence from initial training, with its inclusion in training

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programs and throughout professional life (Llinares, 2013).

To support the development of teacher reflection competence, Llinares (2016) refers to the use of "products of practice" (original Spanish term is "productos de la práctica") or "artifacts of practice" (original Spanish term is "artefactos de la práctica"). An *artifact of practice* can be a video segment that presents the interaction between a group of students and a teacher when they solve a mathematical problem, the productions of students, the sequence of activities used by the teacher to introduce a topic and even their didactic planning.

Products of practice are considered useful tools for the development of the competence of teacher reflection, since they allow to explore and analyze the phenomena that occur during the educational practice, without having the pressure to make decisions quickly; in addition, they offer contexts in which teachers can use their professional knowledge to identify and interpret events, take decisions of action and be more aware of the phenomena that occur in the classroom, as well as to put theoretical ideas into practice, for example, of the didactics of mathematics and subsequently integrate them into the teaching and learning processes in real application contexts (Llinares, 2016).

In the present research, a video segment was used as an *artifact of practice*. The video segment presents the teaching carried out by a secondary school teacher on a contextualized task linked to the mathematical concept of relationship. In particular, the use of video segments that present educational practices allow, based on their analysis, to recognize the complexity of the phenomena that occur during the teaching and learning processes. In addition, its use promotes the ability to identify key aspects of the observed situation and triggers in the observing individual (i.e., student), theoretical references that improve the integration of theoretical and practical knowledge. The use of video also offers the possibility of observing events as many times as desired, which allows identifying situations in the classroom that generally go unnoticed by the teachers themselves (Climent, *et al*., 2013).

Methodology

The research is qualitative and is developed from the interpretive paradigm, constituting itself as a descriptive study. The aim is to understand, interpret and describe the aspects related to mathematical content knowledge and the didactic knowledge of mathematical content, as identified by a group of mathematics teachers in service when analyzing a teaching situation on the concept of relationship. Thus, the problem that arises is practical and, therefore, the investigation must exclusively start from a profound description (Abreu, 2012; Jiménez, 1998). This paradigm is characterized by not being pretentious in generalizing the information, its purpose is to explain the various meanings and facts, observing the behavior of the study subjects; in this way, the behaviors that respond to the research objectives can be described (González-Monteagudo, 2001; Ricoy-Lorenzo, 2006).

Qualitative research follows an inductive process, where it is necessary to explore and describe the phenomenon under study, from the particular to the general. This process is enhanced as the research advances, where the objective is not to test a hypothesis, but to present the results of the data obtained (Hernández, Fernández y Baptista, 2014). Ricoy-Lorenzo (2006) describes the

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qualitative approach as a "phenomenologism (understanding), interested in understanding human behavior from the actual frame of reference from which it acts ..., not generalizable, it remains in the study of isolated cases" (p. 13). Within this order of ideas, it is noted that, among information collection methods, in the realm of qualitative approach, the researcher plays an important role, being an active entity that observes the processes without altering the results, it is an external look at the opinion of the subjects involved.

Particularly, the research problem is approached from case studies. These are pertinent and are adapted to the study of educational reality, based on the analysis of the complexity of a singular case (i.e., a student, a teacher, or a program) to understand their activity in different circumstances (Stake, 1998).

In particular, the research carried out corresponds to an instrumental case study, since "it aims to shed light on some questions or the refinement of a theory. The case [was] selected as typical of other cases [and its] choice…is made to advance the understanding of what interests us" (Buendía, Colás y Fuensanta, 1998, p. 257).

With the analysis of the narratives elaborated by the participating teachers—the cases—information is provided that characterizes the teaching reflection competence, based on the delimitation of the manifestations of knowledge, according to the MTSK model.

Participants

The selection of the teaching staff participating in the study obeys the following conditions: (1) interest and availability to participate in the research, (2) to work in a public or private institution of Secondary Education, (3) to work in a public institution with appointment in a fixed position or with an annual employment contract, (4) have at least five years of professional experience and (5) have taught the content of functions. These conditions are an adaptation of the proposal by Rojas, Carrillo y Flores (2012), for the selection of an expert mathematics teacher. In total, seven teachers who meet these requirements participated, which are identified as P1, P2, P3, P4, P5, P6 and P7.

Information Gathering

The information was collected individually. Each participating teacher received a link to access a virtual site that hosted a video segment and the information collection instrument.

The video segment, approximately six minutes long, shows a teacher introducing the concept of relationship by solving an authentic task. Participants could watch the video as many times as necessary and manipulate it (pause, rewind, forward) according to the reflection process.

For its part, the instrument corresponded to a *narrative guide*. Narrative guides are accounts of a story, where each subject describes what he/she considers important from what is observed; the main function of the narrative is to create, in sequential manner, a relation between the memory and the reflection of a specific topic (Ivars, Fernández y Llinares, 2016). In this case, the narrative guide detailed the indications to analyze the performance of the observed teacher and register personal considerations (ideas, thoughts, points of view), knowledge acquired in their training or expertise, among others. This consisted of two columns: one for the record of the exact time in which the participant observed some aspect considered important

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for reflection, and another for the description of this aspect.

Analysis of the Information

One of the theoretical references of the study is the MTSK model, which proposes a system of categories and subcategories to analyze and characterize the specialized knowledge of mathematics teacher. Said categories and subcategories were adapted to the research topic and coded in units of analysis with which the obtained data were classified, for their subsequent interpretation.

Regarding mathematical content knowledge, the following subdomains were considered:

- *Knowledge of Topics*. It is described from five categories: knowledge about meaning and modes of use, knowledge of properties, knowledge of representations, knowledge of definitions, and procedural knowledge.
- *Structure of Mathematics.* It is described from three categories: enhancer link, cross link, and instrumental link.
- *Practices in Mathematics*. It is described from two categories: mathematical generality and mathematical particularity.

In relation to the didactic knowledge of mathematical content, we consider the following subdomains:

- *Teaching of Mathematics.* It is described from three categories: teaching methodologies, aids for teaching and teaching tasks.
- *Learning of Mathematics.* It is described from four categories: forms of learning, strengths and limitations, student interaction with mathematical content, and student conceptions of mathematics.
- *Mathematics Learning Standards.* It is described from two categories: knowledge of the mathematical contents that are required to be taught and sequences of the topics.

The object of analysis corresponds to each of the reflections that teachers make in their narratives. In this way, for the present research four phases were proposed to organize the analysis of data. The four phases are based on the proposal of Caraballo (2014): (1) *Collection*, in this phase the field work is carried out with the different instruments; for the present study, it consisted of completing the narrative guide, based on what was observed in the class segment. (2) *Organization*, in this phase the data collected in the narrative guides are transcribed, the units of analysis are categorized and each one of them is described. (3) *Processing*, in this phase the content of each unit of analysis is analyzed, and the presentation they are going to have is decided. (4) *Interpretation*, in this phase the patterns within the collected data are identified and findings described.

Once the statements produced by the teaching staff have been categorized, a content analysis is carried out. Bardin (2002) states that in qualitative content analysis small text fragments are studied, wherein the study that is carried out is no longer solely descriptive, but must be taken conscientiously, attempting to apply inference to each obtained data point.

Results

From the perspective adopted for the present study, the analysis carried out is

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based on the proposals on teacher reflection (Llinares, 2016; Mason, 2002) and the MTSK model (Carrillo *et al*., 2014). Hence, based on their teaching experience, the participating teachers identify relevant aspects of a teaching and learning situation in mathematics, which they then interpret to issue affirmations and suggestions—registered in the narratives—to contribute in advancing the performance of the observed teacher. These records provided the information that was analyzed from the context of MTSK categories.

Rooted on what has been presented, the results of the analysis are organized into two sections corresponding to the domains of MTSK knowledge: mathematical content and didactic content. Each of these sections highlights the findings on the subdomains that make up each domain of knowledge.

Mastery of Mathematical Content Knowledge

The information disclosed in the narratives prepared by the participating teachers, which points to the recognition of a mathematical content from the mathematical perspective as a scientific discipline, highlights evidence mainly associated with the knowledge of topics manifested by six participants. As indicated in Table 1, the manifestations associated with knowledge of the structure of mathematics and knowledge of practices in mathematics are identified to a lesser extent.

Next, the main evidence associated with each of the categories defined within the subdomain of mathematical content knowledge are described below. For each of these, textual examples are included with the contributions given by the teachers who participated in the study. These examples are taken from the narratives developed by this group of participants and were presented in a digital format.

Table 1. *Classification of the manifestations provided by the participating teachers according to the subdomains of Mathematical Content Knowledge of MTSK*

Subdomain	P1	P2	P3	P4	P5	P6	P7	Total
Knowledge of Topics (KOT)	1	1	3	3	4	0	0	12
Structure of Mathematics (KSM)	1	0	1	2	1	0	0	5
Practices in Mathematics (KPM)	1	0	0	1	2	2	0	6

Note. P=Participating teacher.

Knowledge of Topics (KOT)

The results obtained regarding mathematical content knowledge in a well-founded way show that participating teachers highlight aspects related to the categories considered, with the exception of the category on properties. Significant differences were found in the frequency with which the participants made comments linked to knowledge of topics (KOT). Next, the results obtained according to the subcategories considered for KOT are presented.

Knowledge about Meaning and Modes of Use

From the analyzed narratives, the participating teachers P2, P3, P4 and P5 identify aspects that can be linked to phenomena associated with the topic (situations), or participants highlight different ways of utilizing and applying the mathematical concept (contexts). In relation to above, two participants highlight the use of the situation in which the problem is immersed. Following are some phrases taken from the narratives that exemplify the ideas raised above.

Participant P4: *"The problem is well contextualized and timely because it derives from everyday life."*

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Participant P5: "The topic of the problem is very good for introducing the basic elements of functions, terms and concepts."

Also, it should be considered that the identification of situations and contexts is carried out implicitly by those who have participated in the study, since they do not mention a specific situation or context of the topic.

Knowledge of Properties

For this category, none of the participating teachers emphasize in their narrative any aspect of the mathematical argumentation of the attributes or qualities of a concept or procedure linked to the topic.

Knowledge of Representations

Regarding the ways of presenting a mathematical concept on the topic, four of the teachers refer to the different systems of representation. In particular, they suggest the use of diverse systems of representation. For example, P4 refers to the use of the iconic representation system, while P1 and P3 suggest the use of the tabular representation system.

Participant P4: "The terms dependent and independent variable, it seems to me that they can be better illustrated with Venn diagrams where everyday situations are shown."

Participant P1: "I would present the exercise in a table that allows analyzing the behavior of the function."

Participant P3: "You can work with a table indicating the additional kilometers and their cost, without saying the word depends."

Finally, it is convenient to note that P5 recognizes in the treated topics (relationship, dependence) the potential they have to promote representations, this by stating "here is a great opportunity to promote the process of representation" when referring to one of the mathematical processes proposed in the mathematics curriculum for Secondary Education in Costa Rica.

Knowledge of Definitions

This analysis did not identify aspects in which participants emphasize a mathematical concept, based on the set of propositions that define it in a precise and unique way, but rather they use the mathematical term—to which the concept refers—to mention some aspect of the analyzed situation.

Procedural Knowledge

In relation to this category, only two participants refer to the use of traditional or alternative algorithms to solve the task shown in the recording. Specifically, P4 identifies an alternative procedure to solve one of the items of the task proposed by the observed teacher; this procedure consisted of subtracting from an available amount of money the fixed cost per kilometer and dividing the result by the cost of each additional kilometer, that is, the student indirectly solves an equation by transposition of terms.

The participating teacher P5 identifies the use of mental calculation, by the observed teacher, when reviewing the answer to one of the sections of the task and, in addition, considers its use relevant; P5 also identifies the alternative "trial and error" procedure used by a student to solve the task and highlights that the observed teacher does not take advantage of this particularity.

Participant P4: "It is very important to give Luis Carlos [a student of the group] the opportunity to explain another way to solve the problem."

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Participant P5: *"I think it is very important for the [the teacher] to employ mental calculation."*
"[the teacher] did not indicate whether this procedure is correct or not."

Structure of Mathematics (KSM)

The manifestations about the relationships between different mathematical concepts that are addressed at the same educational level or at others are found within subdomains with scarce reference; only four participants emphasize any of its components in their narratives. In particular, enhancer and instrumental links are highlighted. The former shows the relationships between concepts to favor their teaching, and the latter promotes the development of other concepts. There is no evidence of cross link in the analyzed narratives.

For example, regarding the enhancer link, P3 and P4 highlight the concept of variable as an element for addressing the association criterion; for its part, P1 refers to the concept of function based on the concept of relationship dealt with. Furthermore, P4 and P5 identify the concept of equation as an instrument that favors the development of the concept of relationship.

Participant P4: *"The definition of the meaning of variable x, I feel that it should have been done before, I consider that it should have occurred in the construction of the association criterion."*
"In the exercise, once the amount of 8000 colones [Costa Rican currency] has been obtained, and after the students have written the association criteria, I consider that the student should be given a space to formulate the equality ..."

Participant P5: *"I think it's very good since it uses skills that students already possess, that is, solving equations which is fundamental in mathematics."*

These contributions show a close link with the experience that these participating teachers have had in the process of teaching mathematics, throughout their professional careers.

Practices in Mathematics (KPM)

In relation to the identification of reasonings that justify proceeding in mathematics as a scientific discipline, little evidence was found in this regard. In particular, only two narratives allude to knowledge related to mathematical generality and three narratives to mathematical particularity. From a general framework, only four of the participating teachers make statements about this knowledge. The results obtained for each of the subcategories considered are shown below.

Mathematical generality

With respect to the aspects related to the general logical structures of mathematical thought, only two participants made comments in this regard: P1 identifies that the observed teacher makes generalizations based on the inputs presented by the students; P6 alludes to a reasoning linked to the way of proceeding in mathematics, associated with the different ways of solving the same exercise.

Participant P6: *"I consider that motivating and teaching that mathematics has different solution methods is very important."*

Mathematical peculiarity

For this indicator, P4, P5 and P6 refer to specific logical structures of thought to address the issue. In general, they indicate reasonings that justify how it is proceeded

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in mathematics—for the concept of relationship—in which models, criteria or formulas that associate variables are designed. For example, P5 highlights the importance of constructing the model (the formula) and the degree of effectiveness of a procedure based on "trial and error".

Participant P5: *"The formula, which corresponds to the criterion, is correct in this case, it is very important for this topic."*
"It is not effective since it could take a long time to reach the expected answer."

On the other hand, P6 coincides with what was indicated by P5, when identifying that a group of students solves the task "by trying and making approximations, until discovering the amount"; the participant adds "as a teacher [P6] I would have said that this way is correct, but it is a bit long and mathematically impractical."

Due to the above, these participants agree on a specific way of proceeding with the topic, by establishing a formula that relates the variables, leading to a better understanding of the situation and the establishment of conclusions about it.

Mastery of didactic knowledge of mathematical content

The subdomains associated with mathematical content knowledge, as an object to be taught and learned, based on established learning standards, acquire significant importance in the present study.

Table 2 emphasizes that the subdomain *teaching of mathematics* is present in all narratives and the demonstrations of its components are significant in most of them. This subdomain is followed by *learning of mathematics*, the evidence of its presence in the narratives is remarkable.

Table 2. *Classification of the manifestations of the participating teachers according to the subdomain of the didactic knowledge of mathematical content of the MTSK*

Subdomain	P1	P2	P3	P4	P5	P6	P7	Total
Teaching of mathematics	7	6	6	12	6	3	1	41
Learning of mathematics	1	0	1	7	3	2	1	15
Mathematics learning standards	0	0	1	0	2	0	0	3

Note. P=Participating teacher.

The results that are linked to the categories defined within the didactic knowledge subdomain of mathematical content are presented below. As in the domain of mathematical content knowledge, these are exemplified with textual extracts taken from the narratives elaborated by the participating teachers.

Teaching of mathematics

Regarding the knowledge of strategies for teaching the concept of relationship, of aids as supports for the teaching of the concept of relationships, and of school mathematics tasks as learning opportunities, the analysis carried out accentuates two categories: 1. Teaching methodologies 2. Aids for teaching. It is to be noted that for this subdomain, expressions were identified that evidence its presence in all the narratives.

Teaching methodologies

The indicators of knowledge that mostly emerge in six of the narratives elaborated by the participating teachers are those relative to teaching methodologies in mathematics. For example, P2, P4, P5 and P6 refer to the strategy used by the observed teacher to solve the proposed task and how

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this can influence student participation during the teaching process. Some of their comments are shown below.

Participant P2: *"The teacher presents the exercise as an equation on the board, but does not stimulate its resolution by the students, mentioning a quick and "recipe-style" solution, leaving aside the formal process of solving equations."*

Participant P4: *"The teacher makes deductions that in my opinion, could, instead of him making them, guide the student to discover them himself."*

Participant P5: *"When giving the floor to the student Luis Carlos, I suppose that he did not want to approach the board, the teacher allows him to provide an explanation from his seat, which I think is very positive, since it encourages participation despite the fact that the young man did not want to pass to the front."*

Participant P6: *"The teacher is putting together a general formula for the posed problem and tells the students that 600 is multiplied by the variable, without allowing them to deduce it."*

On the other hand, problem solving is identified as a teaching theory linked to mathematics education, supported by strategies such as collaborative work and directed questioning.

Participant P5: *"In general, it is very important to work on the topic with the resolution of a situation: a mathematical problem."*

Participant P3: *"It was worked in groups so there is collaborative work among students ...which contributes to the formation of a social subject, capable of transcending individualism, ..."*
"Teamwork among members was demonstrated, since several provided opinions of what they did as a group."

Finally, some of the participants highlight those strategies used by the observed teacher to formalize the concept of relationship, based on the contributions made by the students, and the integration that is made at the end of the lesson of the knowledge developed.

Aids for teaching

The references that participating teachers make about the use of resources or materials for teaching have an important presence in the narratives. The entire group of participants mentions aspects belonging to this category.

Initially, the board is recognized as the available resource (aid) for the observed teacher to develop the lesson. In relation to this idea, the majority of participants point out the efficiency of the resource emphasizing the "correct" use of it, in such a way that the instruction process is favored. In addition, it is suggested the use of other resources such as markers of different colors and paper materials that enrich the teaching of the content and, especially, the resolution of the presented task.

As examples of what has been described, P1, P3 and P6 recognize the board as an aid for teaching; they refer to the efficiency of this aid, indicating the contribution of its use and its effectiveness when they suggest an improvement based on the utilization of other resources to accentuate their contribution.

Participant P3: *"...colors should be used [on the board], to stimulate visualization, at least two colors."*

For their part, the contributions made by P2, P3, P4, P5 and P7 highlight the way in which the observed teacher uses the board as a resource; particularly, they point

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to the order and the posture of the observed teacher in relation to the resource and the group of students.

Participant P4: *"The data from the other groups that remain on the board, by not being taken into account, function more as distractors than as inputs to the explanation."*

Participant P7: *"The board is very messy and I think that is not favorable."*

Tasks for teaching

Regarding the cognitive demands that the observed teacher proposes to the students about the concept of relationship, manifestations of this knowledge have been recognized in three narratives. Additionally, within the *teaching of mathematics* subdomain, this category is the one in which the least evidence is identified. Particularly, participating teachers refer to two aspects of the task. First, P2 and P4 emphasize the relevance and function of the task within the student's learning process.

Participant P2: *"For the next lesson the teacher should return to the same example, to explain the concept of dependent and independent variables, in addition to generating, together with the students, other examples, where they mention and identify the dependent and independent variables."*

Participant P4: *"The problem posed seems appropriate to introduce the subject."*

Second, P3 and P4 focused their attention on the task from the phenomenon that gives it meaning. In this way, they highlight the situation posed in the problem, which brings the task closer to a part of the reality of the students.

Participant P3: *"The situation of the problem is very good to introduce the topic."*

Participant P4: *"The problem is well contextualized..."*

Learning of mathematics

Knowledge about the particularities of learning the concept of relationship is manifested in six analyzed narratives, but with a lower intensity compared to the *teaching of mathematics* subdomain. Of the categories considered in this subdomain, interaction of students with the mathematical content predominates, followed by forms of learning and strengths and limitations.

Interaction of students with the mathematical content

This knowledge, which describes the ways in which students manipulate mathematical content, has been recognized in the diversity of procedures and strategies used by students to solve the task (P1, P5, P6 and P7), and in the arguments and explanations that are proposed as necessary for students to communicate their productions (P4).

Participant P6: *"Some students developed it by testing and making approximations, until they discovered the amount."*

Participant P4: *"It is very important to give Luis Carlos [a student of the group] the opportunity to explain another way to solve what has been proposed."*

Forms of learning

Regarding the knowledge about the forms by which students learn mathematics, P3 and P4 highlight particular styles and strategies with which students appropriate and understand the studied mathematical concept: relation. For example, the following two participating teachers highlight the incidence of the presentation of images and

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the availability of time for the students to execute the resolution of the task.

Participant P3: *"This suggestion [use of a table] is given because it is more visual [for the student], and the result is different from being only auditory."*

Participant P4: *"The student can be given more time to review (mentally and quickly) the knowledge he gained by solving [the task]."*

Also, it is prominent the incidence of an adequate guide by the observed teacher, when the task is solved, in the student's learning process (P4).

Participant P4: *"... the construction of the association criterion must be contrived jointly, teacher and student."*

Strengths and limitations

Three participating teachers refer to the potentialities or obstacles that arise during learning. Regarding the potentialities, conceived as strengths for learning, P5 underlines the use of forms of representation.

Participant P5: *"Here [during the revision of the resolutions of the task] is a great opportunity to encourage the process of representation."*

The obstacles recognized in the narratives of P4 and P6 describe the difficulty that certain procedures have in the topic addressed and the lost opportunity that the observed teacher makes of the diversity of resolutions of the task that the students show, respectively, as limitations for learning.

Participant P4: *"...the formulation of the association criterion... Normally that part is difficult for many [students]."*

Participant P6: *"As a teacher, I would have said that this way is correct [this refers to the resolution shown by a student], but it is a bit long and mathematically impractical. Then, I would address the form of the next group or student that came up with the idea of subtracting the initial 605 and dividing."*

Finally, the expression of the category on the interests and expectations of the students towards mathematics is practically nil; it was only found in the narrative of P5 a reference to the motivation that the observed teacher can promote during teaching, as an element that affects the students' conceptions of this subject and its learning.

Mathematics Learning Standards

From the analysis carried out, this subdomain shows the least number of manifestations in the narratives elaborated by the participating teachers. Only two narratives have recognized descriptions associated with the conceptual organization for the teaching of mathematics at a specific school level, which in the present case corresponds to the fourth year of secondary education in Costa Rica.

In particular, the contributions refer to the usefulness of certain concepts for the development of the topics, within the thematic sequence established in the mathematics curriculum. For example, P5 refers to skills and knowledge previously fostered in the students, which are useful when addressing the concept of relationship: "[the observed teacher] uses skills already possessed by students, the solving of equations which is fundamental in mathematics." In the case of the P3 narrative, the usefulness of the

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topic under development—the concept of relationship—stands out for addressing later topics: “the topic of the problem is very good for introducing the basic elements of functions, terms and concepts.”

Other Contributions Derived from the Analysis of the Narratives

In a complementary way, the analysis has made it possible to identify other ideas that are not directly framed in the subdomains of the MTSK model, since they lack the principles that define the categories that compose them. Some of these ideas are based on general theories of teaching and learning, closer to pedagogy than to the didactics of mathematics. Other ideas point to certain mathematical concepts without referring in a broad and well-founded way to their definition, omitting the propositions that define them and characterize them in a unique and precise way.

For example, P4 indicates the concept of variable; however, his narrative lacks a precise definition of this concept. Similarly, P6 mentions the concept of a dependent variable, P5 expresses the concept of an equation, and P2 refers to the concept of a function. In all cases, there is no clear and precise presentation of propositions that mathematically support that knowledge.

Participant P4: *“The definition of the meaning of variable x, I feel it should have been done earlier.”*

Participant P6: *“[The teacher] points out on the board that C depends on x, a short sentence but it provides a clear idea of the topic they are initiating.”*

Participant P5: *“The teacher equates 8000 with the criteria.”*

Participant P2: *“The teacher must take advantage of the context of the topic, so that students initiate with the concept of function ...”*

On the other hand, P3, P5 and P6 highlight the strategies and methodology used by the observed teacher from a general framework of pedagogy. Participating teacher P7 highlights aspects of the writing of the task, distant from elements such as the purpose, complexity, contextualization, and limitations for its resolution.

In another extent, P2 and P5 focus on the influence that certain elements of the environment (inside and outside of the classroom) exert on student learning.

Participant P2: *“The ambient sound around the classroom can be distracting.”*

Participant P5: *“The teacher must promote silence as respect for the participating student, in addition to generating the ideal atmosphere of attention for the whole class.”*

Finally, it should be noted that in some narratives manifestations about motivational aspects have been identified, which can affect the teaching and learning processes. In detail, P4 highlights that “it is shown that there is trust between the teacher and his students, [the students] do not omit to make a correction on an incorrect data noted by the teacher”, and P5 states “a mark [of approval] that indicates that it [the contribution of students] is correct motivates for future participation.”

Conclusions

The study carried out provides the basis for a series of statements regarding the reflection made by a group of secondary

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school mathematics teachers in service, when they observe a video recording that shows a teacher in a teaching and learning situation, based on solving a task on the concept of relationship.

The teachers, who analyzed this teaching situation on the concept of relationship, show a tendency to identify aspects of the mathematical content from a didactic perspective. In particular, they mobilize their knowledge about the teaching and learning of mathematics, typical of the domain of didactic knowledge of mathematical content (Carrillo, *et al*., 2014).

One of the closest references that these teachers have, to reflect on classroom practice, is their experience as teachers. By way of illustration, it stands out the suggestions they make regarding the use of materials or resources, the references about the usefulness of the concept (or concepts) that are addressed in the lesson, the way in which the observed teacher imparts the class, and the relevance of the diversity of strategies or procedures for solving tasks shown by students.

The teaching reflection carried out by the group of participants is conditioned by the knowledge of the methodological provisions of the mathematics curriculum. For example, when they identify the task that the observed teacher presents to the students to introduce the topic, participants highlight its contextualization and its contribution within the problem-solving methodology, essential components of the Mathematics Study Program (Ministerio de Educación Pública de Costa Rica, 2012). Furthermore, the participating teachers point out some ways of making the concept of relationship present. Obviously, this finding responds to the mathematical process of representing, which is also promoted in Costa Rican mathematics education.

Complementarily, when the participating teachers reflect on a teacher's teaching practice, they highlight elements of teaching and learning theories from a general pedagogical point of view; they refer to mathematical content, but without a timely description of the propositions that define it in a precise and unique manner; and, finally, they project motivation as an essential component of mathematics teaching and learning.

Based on the obtained results, it is recommended that teacher reflection be strengthened as a professional competence of mathematics teachers from initial training and throughout their professional life. In agreement with Llinares (2013), this competence is not innate, it must be developed during the professional training process. It is intended to promote the identification and interpretation of mathematical and didactic particularities of educational practice (one's own and that of other mathematics teachers), to support decision-making relative to the manner in which mathematical concepts are approached during the teaching and learning process in Secondary Education, and to contribute to the professional growth of those who reflect. To accomplish these achievements, it is suggested to carry out activities, inscribed in spaces for individual and group reflection, that implement the use of video recordings or class observation, the elaboration of narratives, and the use of a theoretical reference of the realm of didactics of mathematics as the foundation of the process of reflection.

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de los conceptos básicos de función desde la mirada profesional de situaciones de enseñanza" translated as "Specialized knowledge of mathematics teachers for teaching the basic concepts of function from the professional perspective of teaching situations."

Conflict of interests

The authors declare that they have no conflict of interest.

Authors' declaration of contribution

The total percentage of contribution for the conceptualization, preparation and correction of this article was the following: MPA. 34 %, JRLF. 33 % y JEG. 33 %. All authors affirm that the final version of this article was read and approved.

Declaration of data availability

Data supporting the results of this study will be made available by the corresponding authors **[MPA, RLF or JEG]**, upon reasonable request.

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